Comment on "Black hole constraints on varying fundamental constants"

V.V. Flambaum
School of Physics, The University of New South Wales, Sydney NSW 2052, Australia
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In the Letter [1] (also [2]) there is a claim that the generalised second law of thermodynamics (entropy increase) for black holes provides some limits on the rate of variation of the fundamental constants of nature (electric charge e, speed of light c, etc.). We have come to a different conclusion. The results in [1, 2] are based on assumption that mass of a black hole does not change without radiation and accreation. We present arguments showing that this assumption is incorrect and give an estimate of the black hole mass variation due to $\alpha = e^2/\hbar c$ variation using entropy (and quantum energy level) conservation in an adiabatic process. No model-independent limits on the variation of the fundamental constants are derived from the second law of thermodynamics.

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It is convenient to present the dimensionless entropy of a charged black hole [3] in terms of dimensionless parameters:

$$S = \pi[\mu + \sqrt{\mu^2 - Z^2\alpha}]^2 \tag{1}$$

Here $\mu = M/M_P$, M is the black hole mass,$M_P = (\hbar c/G)^{1/2}$ is the Plank mass, Ze is the black hole charge, the Boltzmann constant $k = 1$ (for a rotating black hole $S = \pi([\mu + \sqrt{\mu^2 - Z^2\alpha - J(J+1)/\mu^2}]^2 + J(J+1)/\mu^2))$. This expression does not contain explicitly any parameters which have dimension (speed of light, proton electric charge, etc.). Therefore, one may only discuss variation of two dimensionless parameters: mass of the black hole in units of the Plank mass (μ) and α. In all known adiabatic processes the entropy S is conserved. It is natural to assume that this is also valid for a slow variation of the fundamental constants. Then eq. (1) gives $\mu(t)$ in terms of $\alpha(t)$ and constant S:

$$\mu = \frac{M}{M_P} = \frac{(S/\pi) + Z^2\alpha}{2\sqrt{(S/\pi)}} \tag{2}$$

The event horizon area A of the black hole is quantized [4]. Because of the relation between the entropy S and the horizon area A we obtain the entropy quantization $S = (c^3/4G\hbar)A = \pi\gamma\cdot n$, where γ is a numerical constant, n is an integer. This gives us μ as a function of α:

$$\mu = \frac{M}{M_P} = \frac{\gamma\cdot n + Z^2\alpha}{2\sqrt{\gamma\cdot n}} \tag{3}$$

One may compare this result with that for the hydrogen atom where we have the electron energy levels $E_n/m_e c^2 \approx 1 - \alpha^2/2n^2$. An adiabatic variation of parameters do not cause any transitions between non-equal levels, therefore the variation of atomic and black hole masses is given by the stationary formulas with α depending on time. Note that the variations of masses do not contradict to the energy conservation law since atoms and black holes are not closed systems, they are interacting with the Universe. Indeed, in theoretical models the variation is driven by an evolution of some scalar field, and energy of this field must be taken into account. The entropy in the adiabatic case does not change. Therefore, the time dependence of α does not lead to any specific problems for the black holes, it just gives us the dependence $\mu(t)$.

One could suggest (see e.g. [2]) that the variation $\alpha(t)$ may lead to a negative expression under the square root sign in eq. (1) (which also appears in the formulas for the horizon area and temperature), and this may give a limit on the allowed variation. Here again an atomic analogy may be useful. The energy of the $1s$ level in the Dirac equation is $E = mc^2\sqrt{1 - Z^2\alpha^2}$. If we increase α beyond $1/Z$ the expression under the square root becomes negative and the stationary state $1s$ disappears. This only means that the system becomes non-stationary. The strong Coulomb field creates an electron-positron pair, the positron goes to infinity and the nuclear charge reduces to a sub-critical value (Z to $Z-1$). Therefore, the possibility of a negative expression under the square root sign in the electron energy level or eq. (1) for the black hole entropy does not mean that certain variations of α are forbidden.

An inclusion of the Hawking radiation and accreation leads to an increase of entropy, therefore, it does not violate the second law of thermodynamics, at least for the adiabatic variation of the fundamental constants. For a non-adiabatic variation one should accurately take into account contributions of scalar fields driving the variation.

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